The AIMer Signature Scheme

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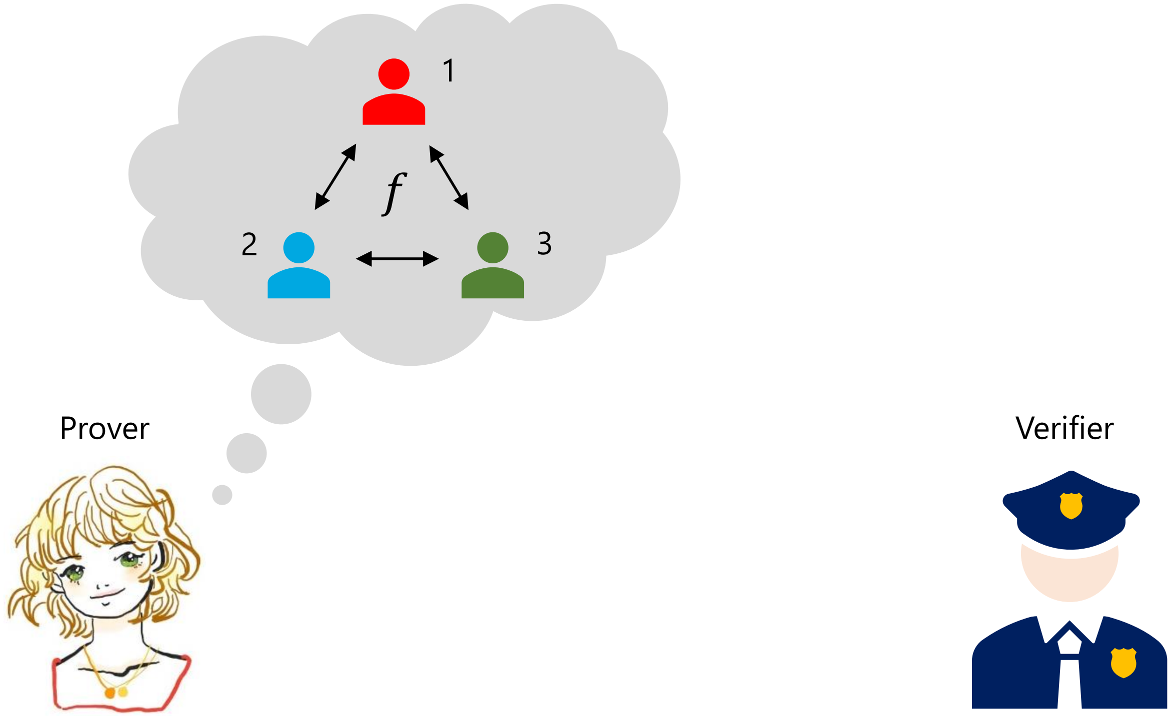

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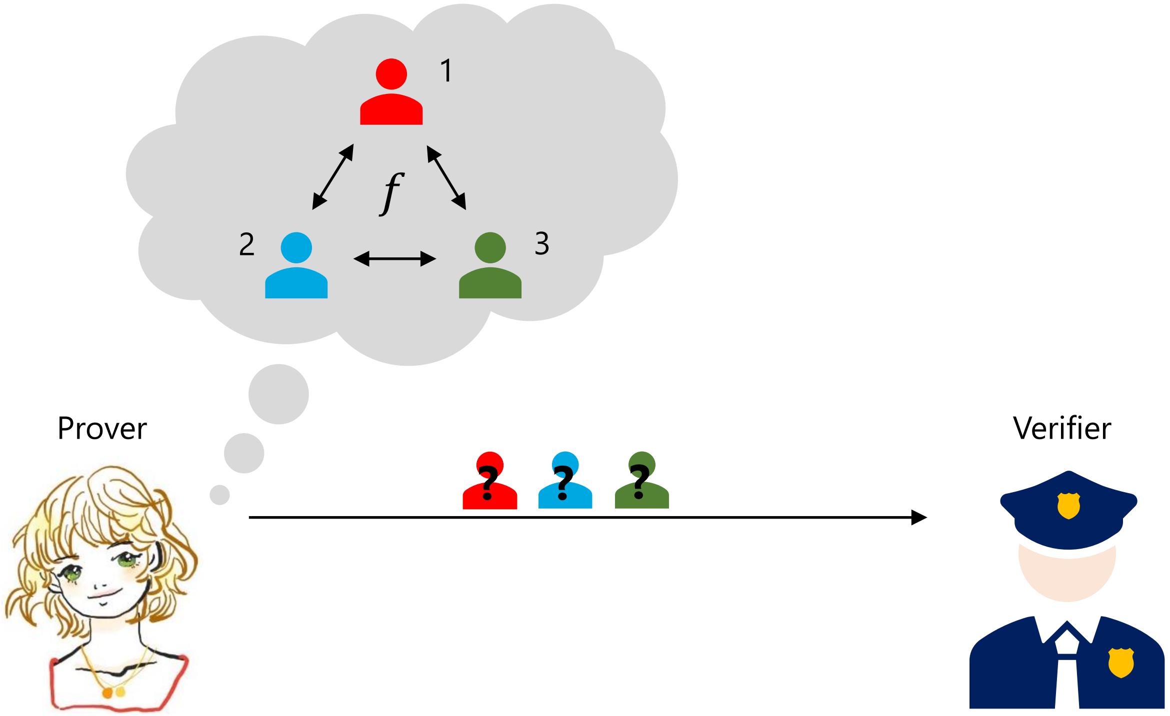

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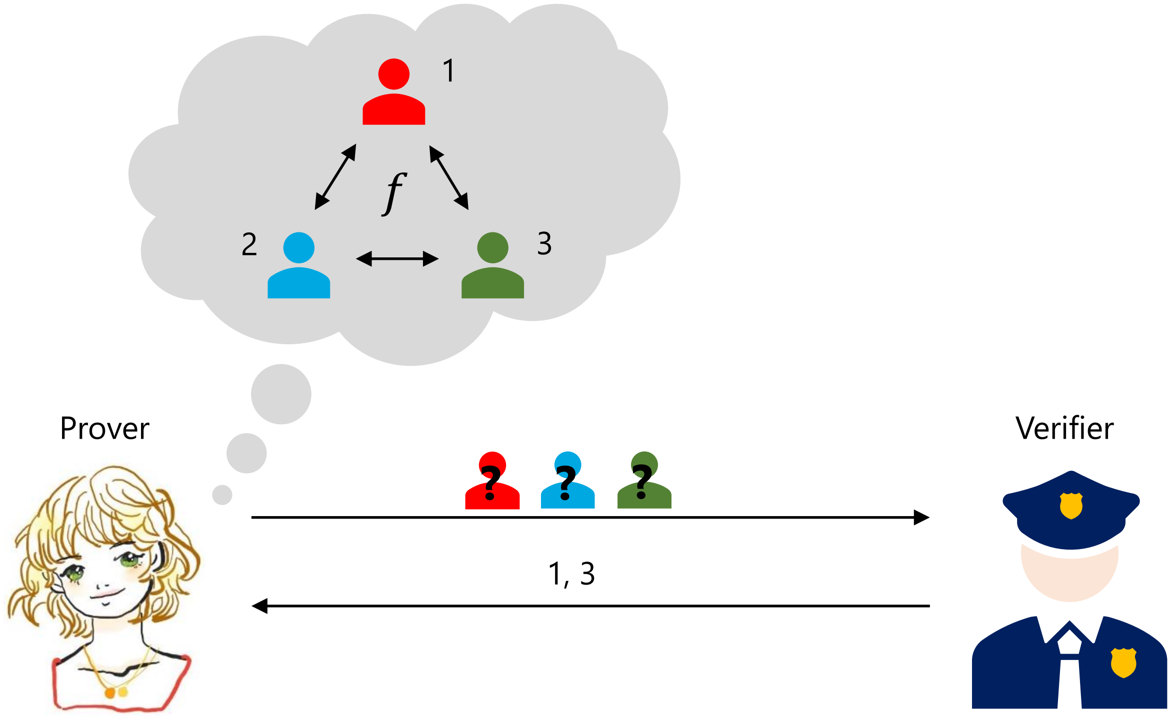

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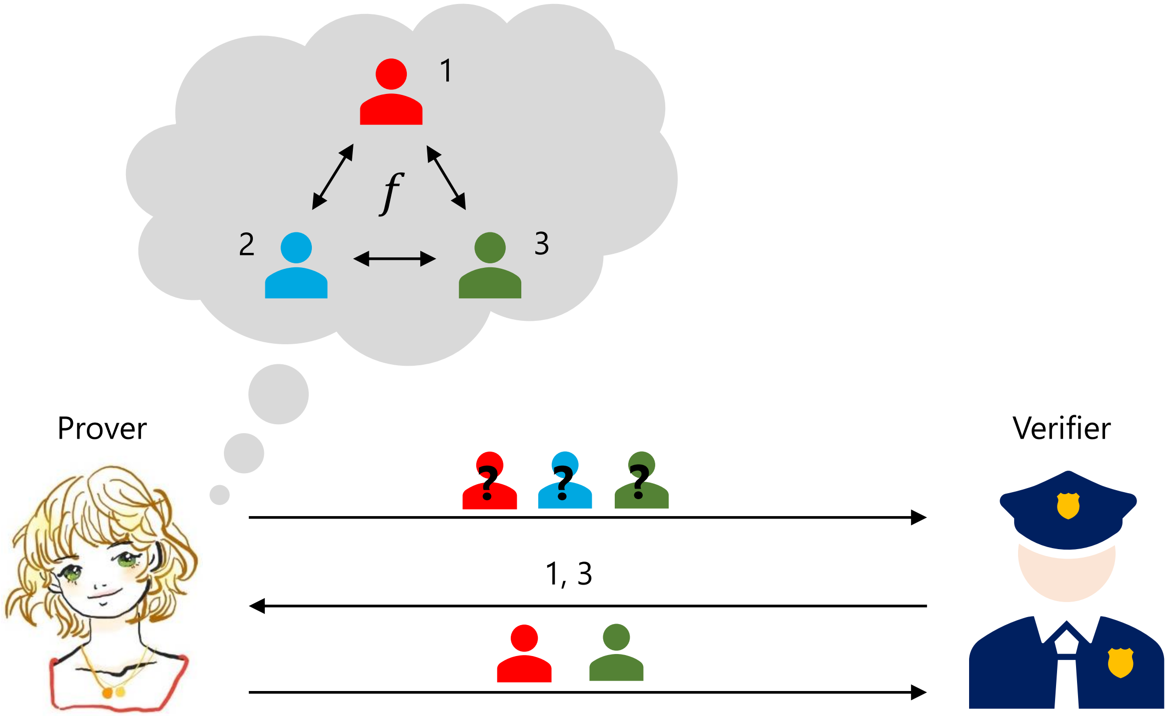
    4. Prover opens the challenged view
    5. Verifier checks consistency



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- Requires only random oracle and one-way function
- Sacrificing-based inner product check
  - Wants to check multiplication triples $\{(x_i, y_i, z_i)\}_i$ such that $x_i \cdot y_i = z_i$
  - Inner product triple $\left((a_i, y_i), c\right)$ such that $\sum_i a_i y_i = c$
  - For random $\{\varepsilon_i\}_i$,

$$[\alpha_i] = \varepsilon_i \cdot [x_i] + [a_i]$$

Open α_i

$$[v] = \sum_i (\alpha_i [y_i] - \varepsilon_i [z_i]) + [c]$$

Check $v = 0$

  - Soundness = $1/|\mathbb{F}|$

Efficient Circuit for BN++

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  - $x_1 \cdot y = z_1$, $x_2 \cdot y = z_2$
- Known output share
  - If an output of a multiplication is already known, then the signer can save the signature size
  - E.g., $y = x^{-1} \rightarrow xy = 1$, 1 is known without any computation

Symmetric Primitive AIM

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Sufficient security against algebraic attacks + Best performance when combined to BN++

Inverse S-box

- Inverse S-box ($x \mapsto x^{-1}$) is widely used in MPC/ZKP-friendly ciphers
  - High degree, but quadratic relation ($xy = 1$)
  - Invertible
  - Nice DC/LC resistance
  - But, produces many linearly independent quadratic equations

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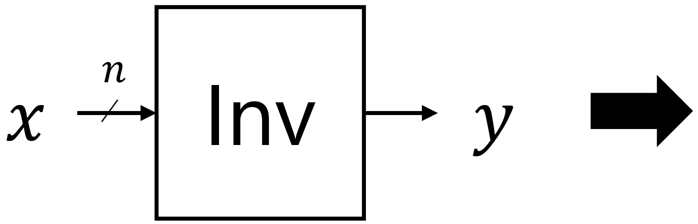

$$\begin{cases} f_1(x_1, \dots, x_n, y_1, \dots, y_n) = 0 \\ \vdots \\ f_{5n}(x_1, \dots, x_n, y_1, \dots, y_n) = 0 \end{cases}$$

$5n$ quadratic equations

c.f. optimally n equations

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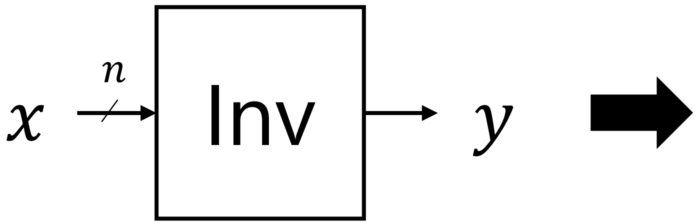

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More equations lead to a weaker resistance against algebraic attacks!

Candidates of Appropriate S-box

- Niho exponent
  - $x \mapsto x^{2^s+2^{s/2}-1}$ over $\mathbb{F}_{2^n}$, $n = 2s + 1$
  - n equations, high-degree
  - 2 multiplications, odd-length field
- NGG exponent (Nawaz et al., 2009)
  - $x \mapsto x^{2^{s+1}+2^{s-1}-1}$ over $\mathbb{F}_{2^n}$, $n = 2s$
  - $2n$ equations, even-length field, good DC/LC resistance
  - 2 multiplications
- Mersenne exponent
  - $x \mapsto x^{2^s-1}$ over $\mathbb{F}_{2^n}$
  - $3n$ equations, even-length field, single multiplication
  - moderate DC/LC resistance
- Gold exponent
  - $x \mapsto x^{2^s+1}$ over $\mathbb{F}_{2^n}$
  - Even-length field, single multiplication, good DC/LC resistance
  - $4n$ equations

Repetitive Structure for BN++

- Repeated multiplier technique (in BN++)
  - If prover needs to check multiple multiplications with a same multiplier,
    - e.g., $x_1 \cdot y = z_1, x_2 \cdot y = z_2$
  - Then, the prover can prove them in a batched way
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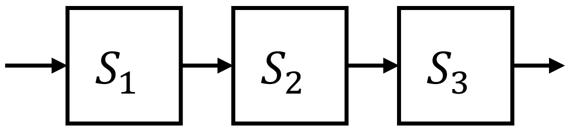
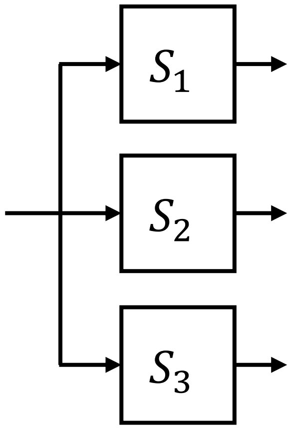
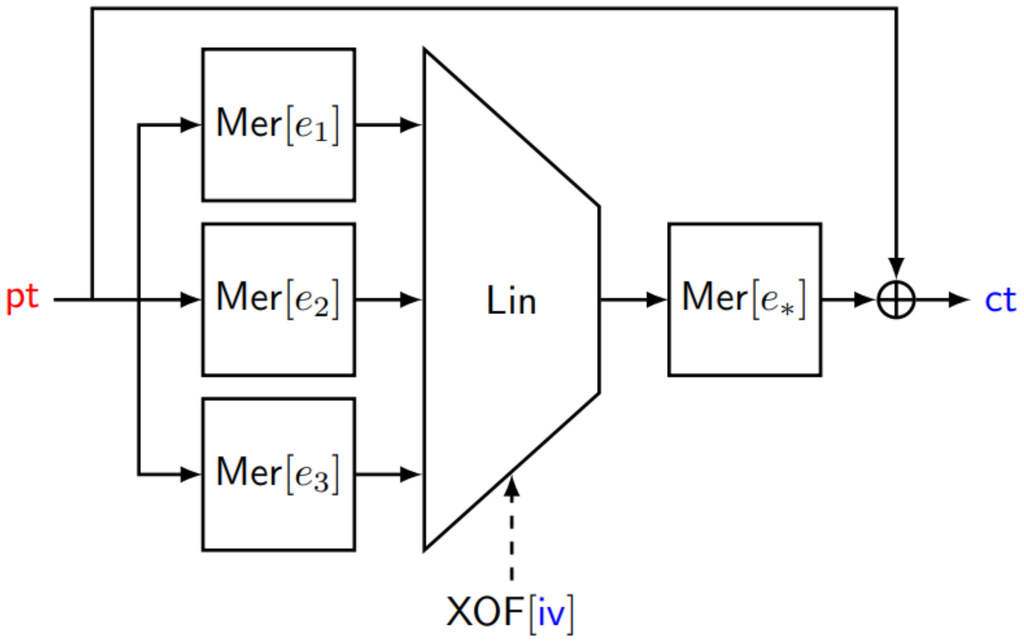


Serial S-box
(Limited application of repeated multiplier)



Parallel S-box
(Full application of repeated multiplier)

Symmetric Primitive AIM



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  - Invertible, high-degree, quadratic relation
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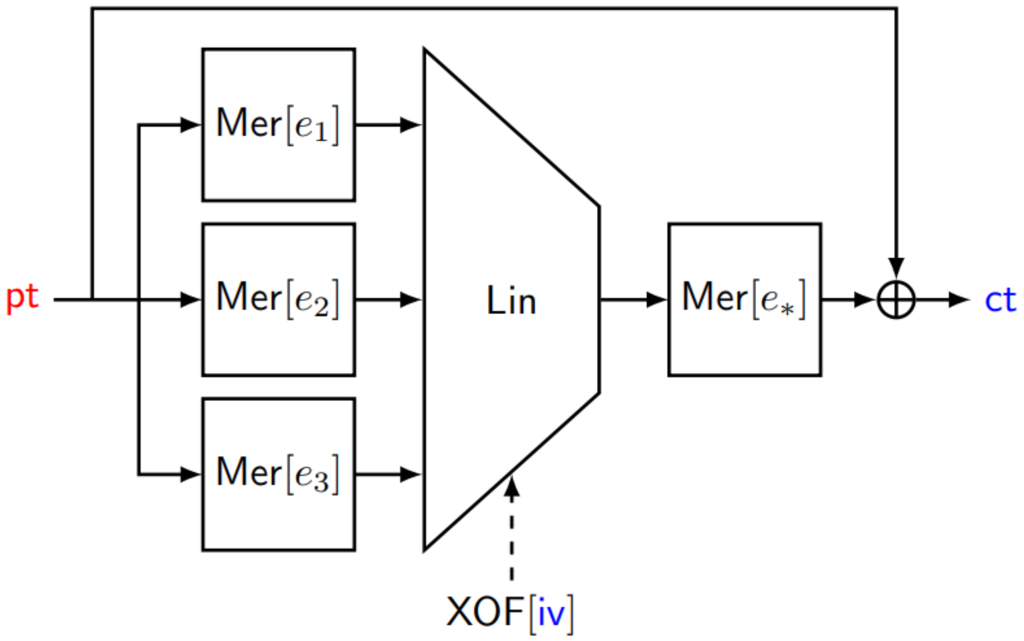

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  - Feed-forward construction
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  - Locally-computable output share

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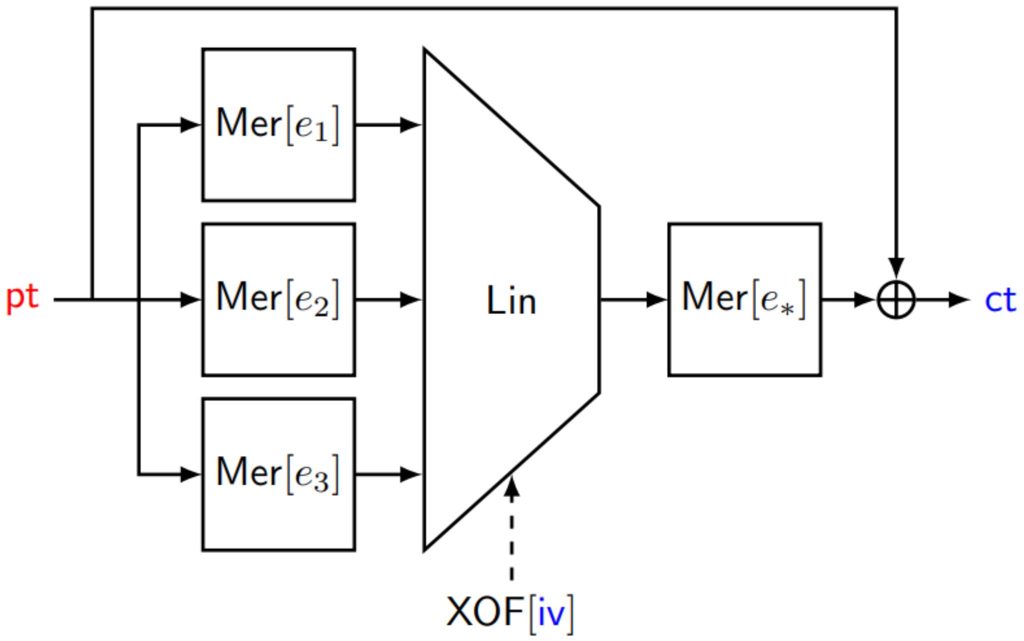

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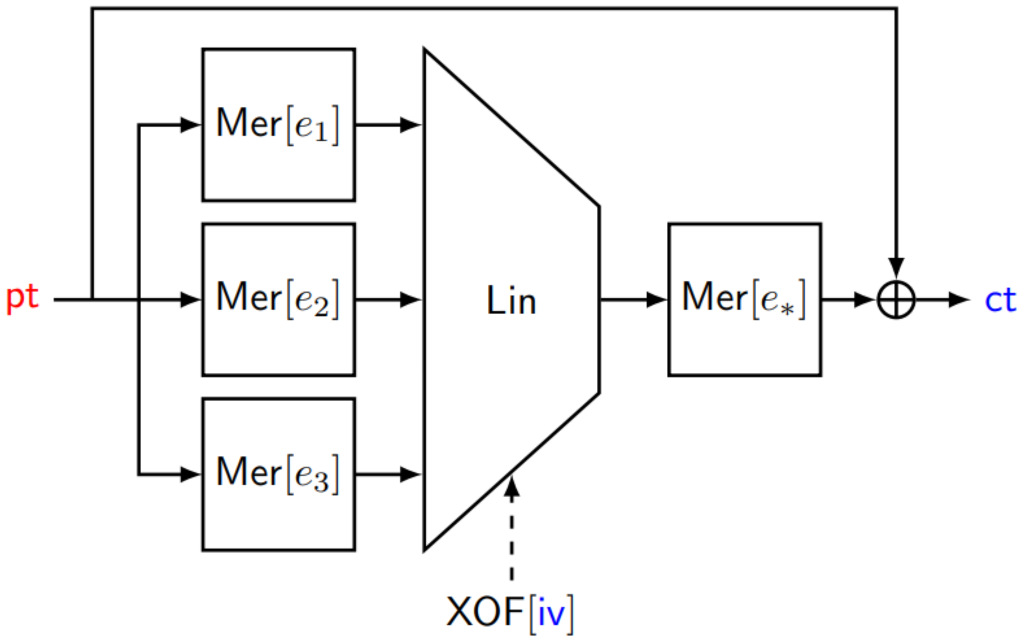

Scheme	λ	n	ℓ	e_1	e_2	e_3	e_*
AIM-I	128	128	2	3	27	-	5
AIM-III	192	192	2	5	29	-	7
AIM-V	256	256	3	3	53	7	5

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Analyses on AIM

Recent Analysis on AIM (Jul. 24)

- Fukang Liu and Mohammad Mahzoun proposed a fast exhaustive search attack on AIM*
- It achieves 10-12 bits smaller complexity compared to brute-force attack on AIM
- The main vulnerability was that there are low-degree equations with n Boolean variables
- Increasing exponents resolves this vulnerability

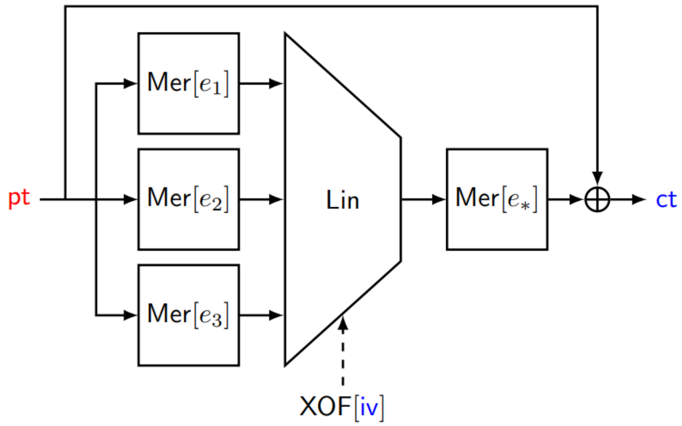
* F. Liu and M. Mahzoun. "Algebraic Attacks on RAIN and AIM Using Equivalent Representations". Cryptology ePrint Archive. Report 2023/1133

Recent Analysis on AIM (Jul. 27)

- Liu introduce another possible vulnerability to our team*
- Setting a new variable $w = \mathrm{pt}^{-1}$ leads to easier system than expected
  - AIM is claimed to be secure under an ℓn-variable system with $3\ell n$ quadratic equations
  - A $2n$-variable system including $5n$ quadratic equations and $5n$ cubic equations

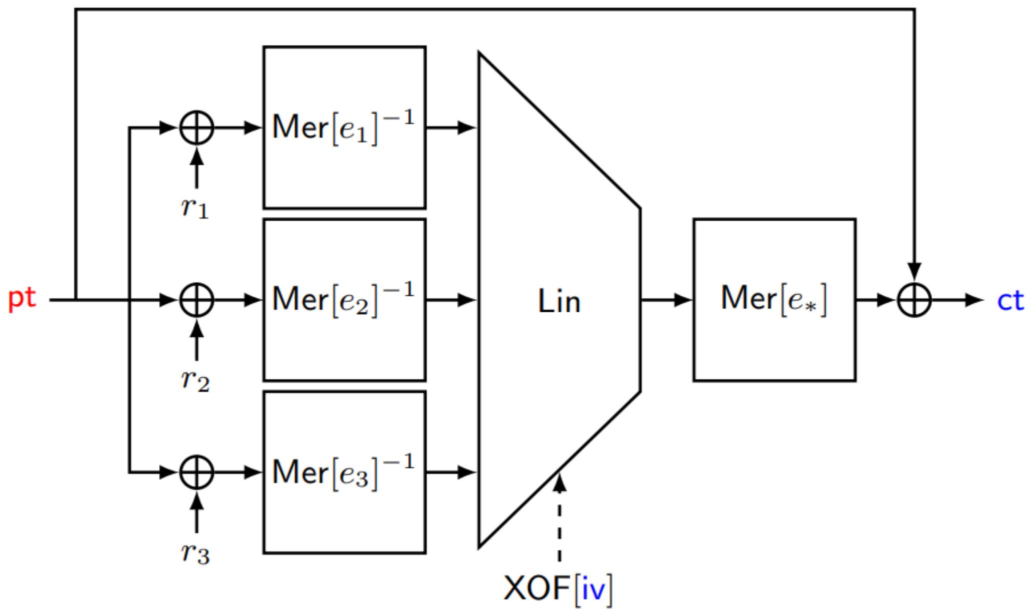
$$\begin{cases} \mathrm{pt} \cdot w = 1 \\ \mathrm{Lin}\left(\mathrm{pt}^{2^{e_1}}w, \mathrm{pt}^{2^{e_1}}w, \mathrm{pt}^{2^{e_1}}w\right) \cdot (\mathrm{pt} + \mathrm{ct}) = \mathrm{Lin}\left(\mathrm{pt}^{2^{e_1}}w, \mathrm{pt}^{2^{e_1}}w, \mathrm{pt}^{2^{e_1}}w\right)^{2^{e_*}} \\ \mathrm{Lin}\left(\mathrm{pt}^{2^{e_1}}w, \mathrm{pt}^{2^{e_1}}w, \mathrm{pt}^{2^{e_1}}w\right) \cdot (1 + w \cdot \mathrm{ct}) = w \cdot \mathrm{Lin}\left(\mathrm{pt}^{2^{e_1}}w, \mathrm{pt}^{2^{e_1}}w, \mathrm{pt}^{2^{e_1}}w\right)^{2^{e_*}} \end{cases}$$

- Note that this attack is **not practically feasible** on AIM



* In private communication

AIM2: Secure Patch for Algebraic Attacks (In Progress)



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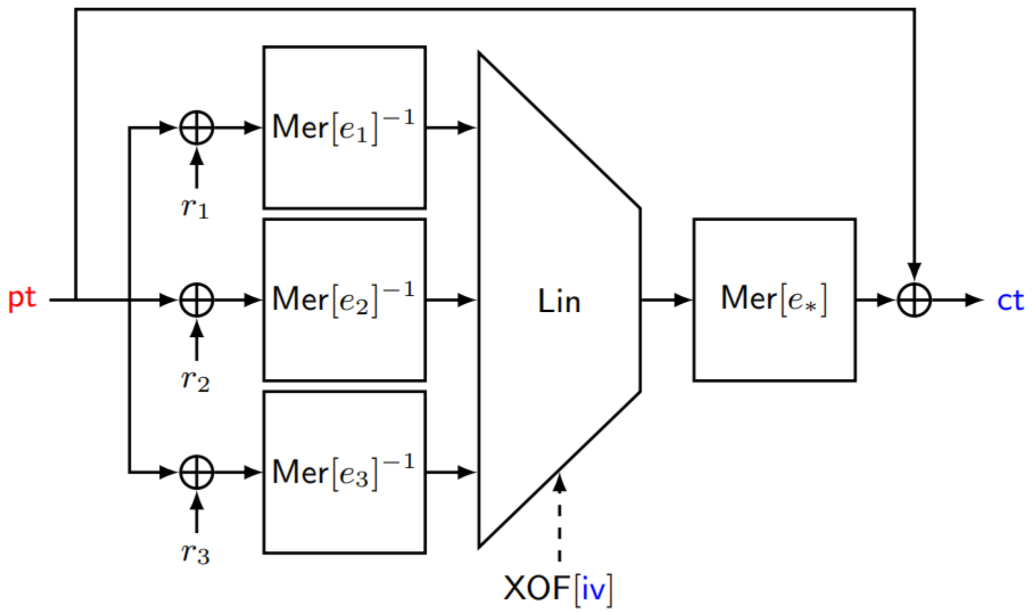

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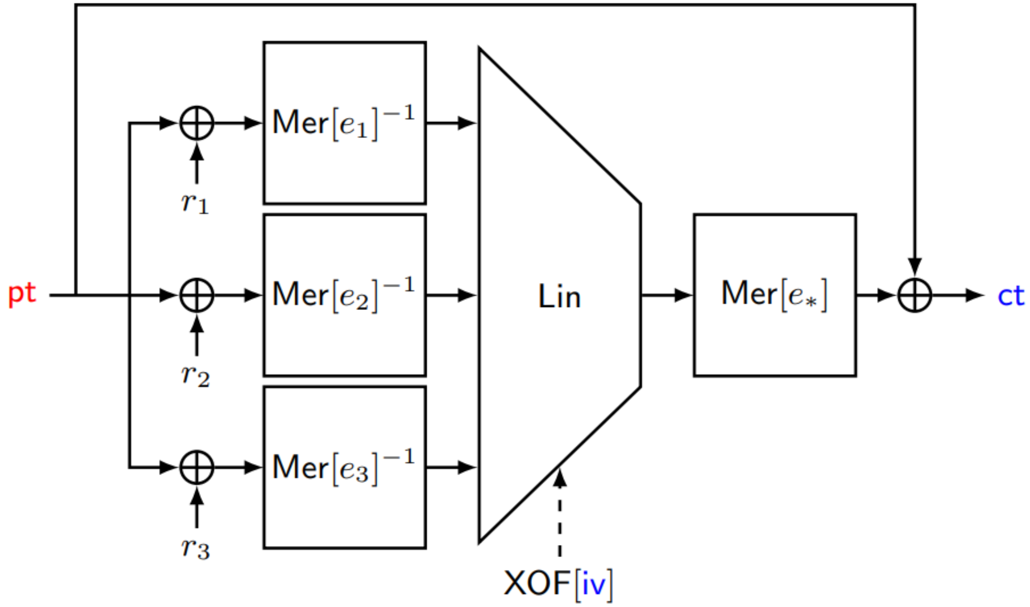

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  - To mitigate fast exhaustive search
- Fixed constant addition
  - To differentiate inputs of S-boxes
  - Increase the degree of composite power function

$$(x^a)^b \text{ vs } (x^a + c)^b$$

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- Preliminary version can be found in our website!

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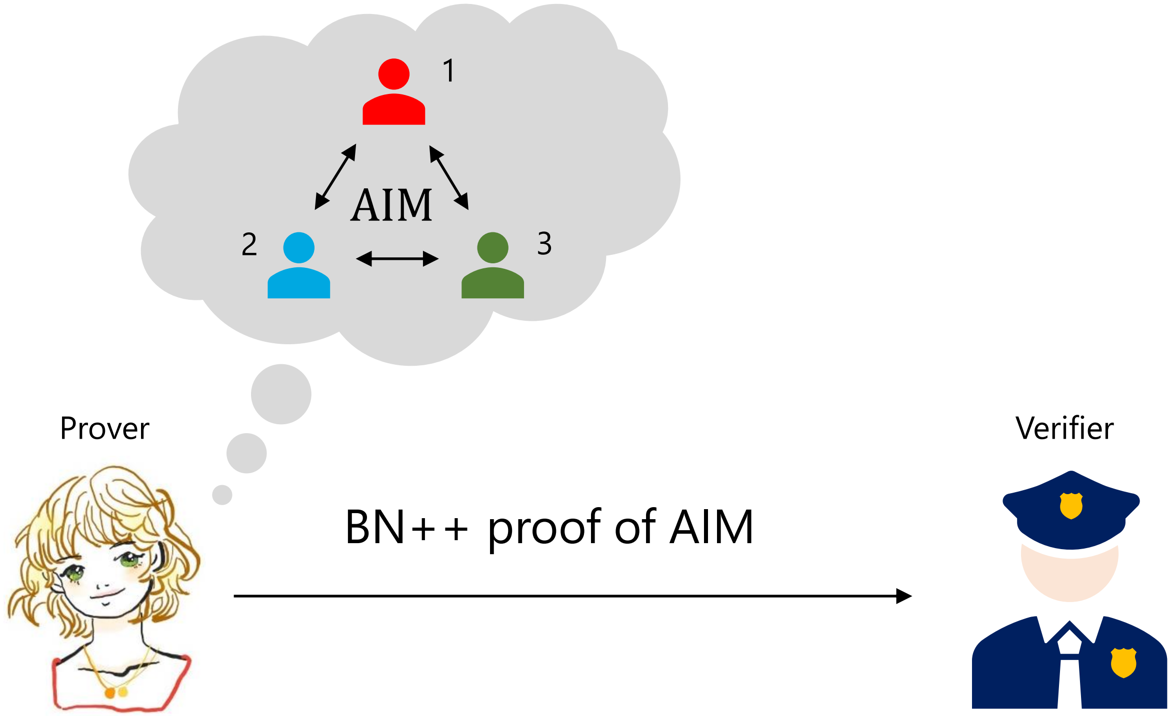

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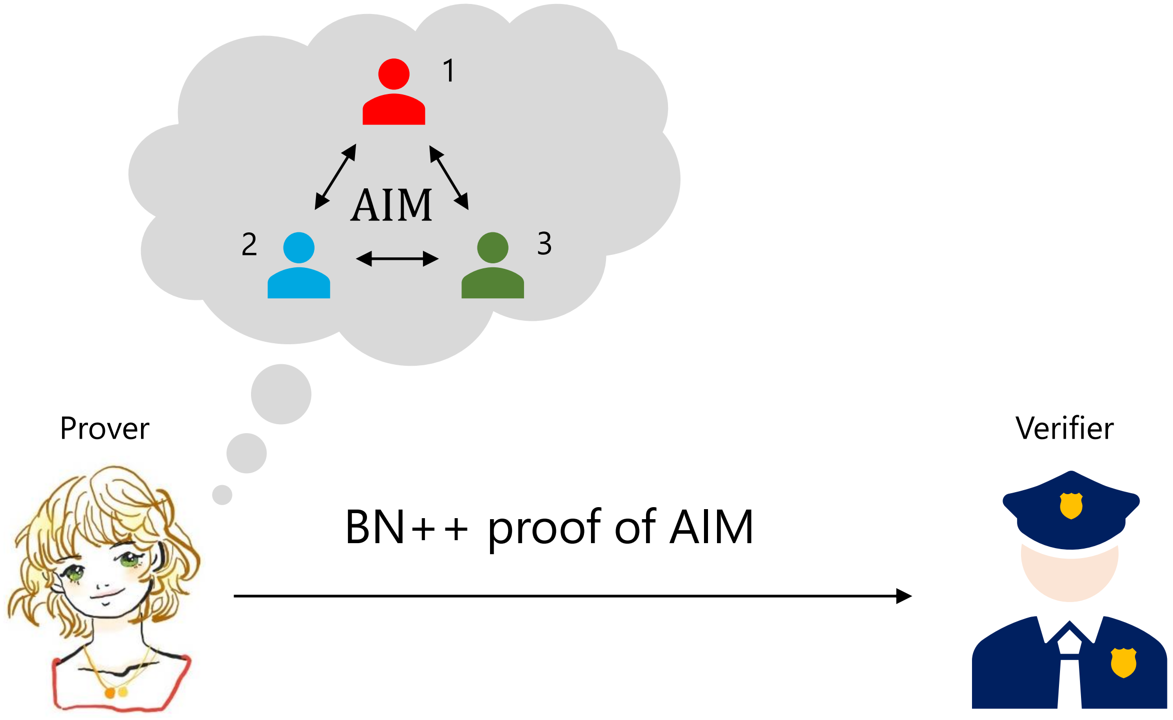
    - Trade-offs between signatures size and speed
    - Randomness misuse resistance



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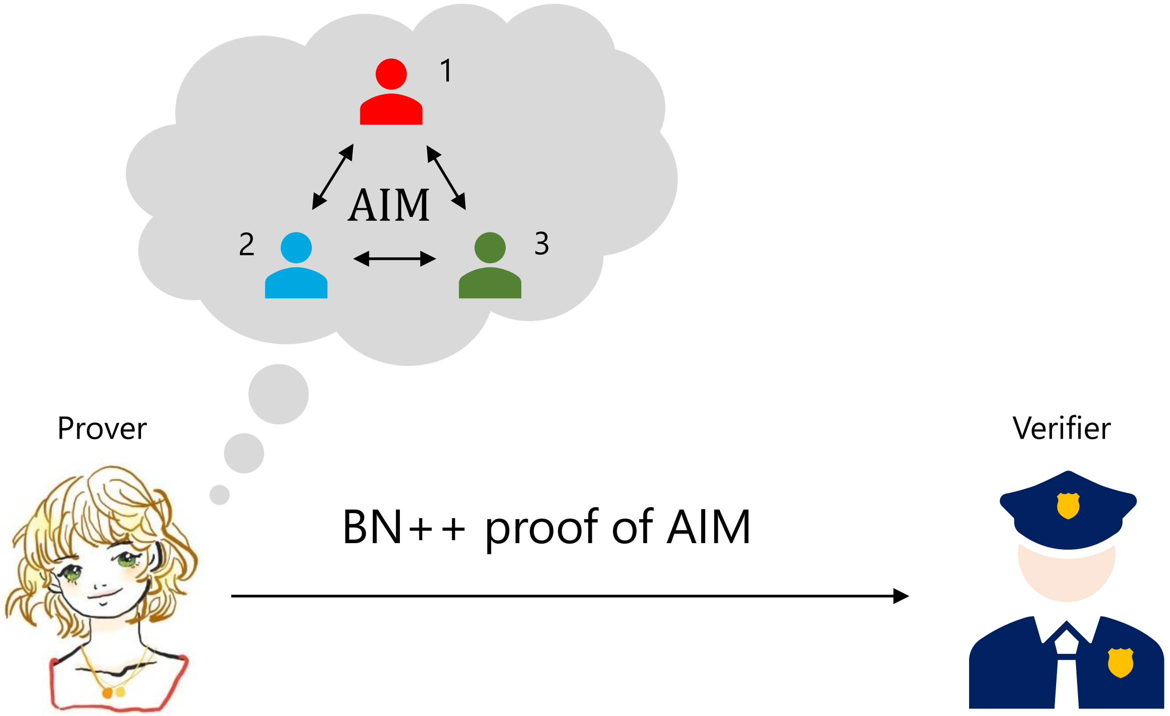
- Limitations
  - Newly-designed symmetric primitive AIM
  - Moderately large signature size (3.8~5.9 KB)
  - Slow signing/verifying speed (0.59~22 ms)



Performance Comparison

Scheme	pk (B)	sig (B)	Sign (ms)	Verify (ms)
Dilithium2	1312	2420	0.10	0.03
Falcon-512	897	690	0.27	0.04
SPHINCS$^+$-128s	32	7856	315.74	0.35
SPHINCS$^+$-128f	32	17088	16.32	0.97
Picnic1-L1-full	32	30925	1.16	0.91
Picnic3	32	12463	5.83	4.24
Banquet	32	19776	7.09	5.24
Rainier$_3$	32	8544	0.97	0.89
BN++Rain$_3$	32	6432	0.83	0.77
AIMer-L1 (Not updated)	32	5904	0.59	0.53
AIMer-L1 (Not updated)	32	3840	22.29	21.09

Some Remarks

- Remark
  - We submitted AIMer to KpqC and NIST PQC competition
  - Our homepage: https://aimer-signature.org
  - We are waiting for **third-party analysis**!

- Future work
  - Updates on the specification document
  - QROM security of AIMer
  - More optimization on BN++

Thank you!
Check out our website!